Quantitative Literacy:
Thinking Between the Lines

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Chapter 1:
Critical Thinking

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Chapter 1 Critical Thinking

Lesson Plan

- Public policy and Simpson's paradox: Is "average" always average?
- Logic and informal fallacies: Does that argument hold water?
- Formal logic and truth tables: Do computers think?
- Sets and Venn diagrams: Pictorial logic
- Critical thinking and number sense: What do these figures mean?

1.5 Critical thinking and number sense: What do these figures mean?

Learning Objective:

- Cope with the myriad measurements the average consumer encounters every day.
  - Magnitudes
  - Taming large and small numbers
  - Estimation

1.5 Critical thinking and number sense: What do these figures mean?

- **Examples of powers of 10**
  - Positive powers of 10

$10^3 = 1{,}000$ (three zeros) is a thousand.

$10^6 = 1{,}000{,}000$ (six zeros) is a million.

$10^9 = 1{,}000{,}000{,}000$ (nine zeros) is a billion.

$10^{12} = 1{,}000{,}000{,}000{,}000$ (twelve zeros) is a trillion.

  - Negative powers of 10

$10^{-2} = 0.01$ is a hundredth.

$10^{-3} = 0.001$ is a thousandth.

$10^{-6} = 0.000\ 001$ is a millionth.

$10^{-9} = 0.000\ 000\ 001$ is a billionth.

1.5 Critical thinking and number sense: What do these figures mean?

Quick Review Exponents

Negative exponents: a^{-n} is the reciprocal of a^n.

Definition	Example
$a^{-n} = \frac{1}{a^n}$	$10^{-3} = \frac{1}{10^3} = \frac{1}{1000} = 0.001$

Zero exponent: If $a \neq 0$, then $a^0 = 1$.

Basic properties of exponents:

Definition	Example
$a^p a^q = a^{p+q}$	$10^2 \times 10^3 = 10^{2+3} = 10^5 = 100{,}000$
$\frac{a^p}{a^q} = a^{p-q}$	$\frac{10^6}{10^4} = 10^{6-4} = 10^2 = 100$
$(a^p)^q$	$(10^3)^2 = 10^{3\times 2} = 10^6 = 1{,}000{,}000$

1.5 Critical thinking and number sense: What do these figures mean?

- **Example (Using powers of 10):** In the 1980s, one of the book's authors owned a microcomputer that had a memory of 64 kilobytes. The computer on his desk today has 4 gigabytes of memory. Is the memory of today's computer tens, hundreds, or thousands of times as large as the old computer's memory?

- **Solution:**

$$\frac{\text{New memory size}}{\text{Old memory size}} = \frac{4 \times 10^9 \text{bytes}}{64 \times 10^3 \text{bytes}} = \frac{4}{64} \times 10^{9-3} = 62{,}500.$$

The new computer has over 60 thousand times as large a memory as the old computer.

1.5 Critical thinking and number sense: What do these figures mean?

- **Example (Understanding large numbers):** As of January 2009, the national debt was about 10.6 trillion dollars, and there were about 305 million people in the United States. The national debt is not just an abstract figure. The America people actually owe it. Determine how much each person in the United States owes.

- **Solution:** Use powers of 10 to express:

  the national debt : 10.6 trillion $= 10.6 \times 10^{12}$

  the population of the United States: 305 million $= 305 \times 10^{6}$

$$\text{Debt per person} = \frac{10.6 \times 10^{12}}{305 \times 10^{6}} = \frac{10.6}{305} \times 10^{6} = 0.0348 \times 10^{6}$$

Each person owes 0.0348×10^{6} dollars $= \$34{,}800.$

1.5 Critical thinking and number sense: What do these figures mean?

- **Example (Understanding very small numbers):** Human hair can vary in diameter, but one estimate of the average diameter is 50 microns or 50×10^{-6} meter. How many 10-nanometer particles could be stacked across the diameter of a human hair?
- **Solution:** 10 nanometers is 10×10^{-9} meter

$$\frac{\text{Diameter of hair}}{\text{Diameter of nanoparticle}} = \frac{50 \times 10^{-6}\text{meter}}{10 \times 10^{-9}\text{meter}} = 5 \times 10^{3}$$

Thus, 5000 of the nanoparticles would be needed to span the diameter of a human hair.

1.5 Critical thinking and number sense: What do these figures mean?

- **Example (Estimating costs):** You are traveling to Canada, and you wonder whether to gas up before you cross the border. You see a sign at a gas station on the U.S. side of the border showing $3.77, and on the Canadian side of the border, you see a sign showing $1.10. One might think that the Canadian gas is much cheaper, but let's use critical thinking and look a little closer.

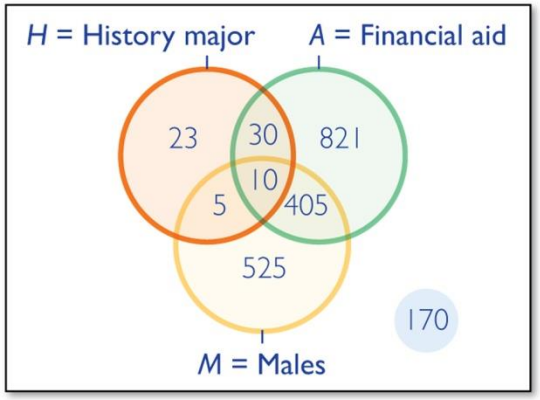

FIGURE 1.23 Students.

In Canada, gasoline is measured in liters rather than gallons. Also, the dollar sign refers not to U.S. dollars but to *Canadian* dollars.

The sign on the Canadian side of the border means that gasoline costs **1.10** *Canadian dollars per liter.*

1.5 Critical thinking and number sense: What do these figures mean?

- **Example (cont.):** We use an exchange rate of 0.94 U.S. dollar per Canadian dollar.

1. Use the fact that a quart and a liter are nearly the same and that the Canadian dollar is worth a little less than the U.S. dollar to get a quick estimate of the cost of gasoline in Canada measured in U.S. dollars per gallon.
2. There are 3.79 liters in a gallon. Use this fact to find the actual cost in U.S. dollars per gallon of gasoline at the Canadian station. Was your estimate in part 1 good enough to tell you where you should buy your gas?

1.5 Critical thinking and number sense: What do these figures mean?

- **Solution:** 1. 1 Canadian dollar is about 1 U.S. dollar and 1 liter costs 1.10 Canadian dollars,

  gas costs about 1 U.S. dollar per liter.

  A quart is about a liter, there are about 4 liters in a gallon,

  gas in Canada costs about 4 U.S. dollars per gallon.

  Thus, gas is cheaper at the station in the United States.

2. There are 3.79 liters in a gallon; gasoline in Canada costs

$$3.79\frac{\cancel{\text{liters}}}{\text{gallon}} \times 1.10\frac{\text{Canadian dollars}}{\cancel{\text{liters}}} = 4.169\frac{\text{Canadian dollars}}{\text{gallon}}$$

Each Canadian dollar is worth 0.94 U.S. dollars.

$$\text{Cost in U. S. dollars} = 4.169 \times 0.94 = 3.92.$$

The actual cost of gas on the Canadian side of the border is 3.92 U.S. dollars per gallon.

1.5 Critical thinking and number sense: What do these figures mean?

- **Example (Areas):** Carpet is priced in both square feet and square yards. There are 3 feet in a yard. How many square feet are in a square yard?
- **Solution:**

Recall that the area of a rectangle is the length times the width. So the area in square feet of a square yard is:

$$\begin{aligned}&\text{Area of 1 square yard}\\&= \text{Length} \times \text{Width} = 3 \text{ feet} \times 3 \text{ feet}\\&= 9 \text{ square feet}\end{aligned}$$

1.5 Critical thinking and number sense: What do these figures mean?

- **Example (Volumes):** There are 3 feet in a yard. How many cubic feet are in a cubic yard?
- **Solution:**

Recall that the volume of a box is the length times the width times the height. So the volume in cubic feet of a cubic yard is:

$$\begin{aligned}&\text{Volume of 1 cubic yard}\\ &= \text{Length} \times \text{Widt} \times \text{Height}\\ &= 3 \text{ feet} \times 3 \text{ feet} \times 3 \text{ feet}\\ &= 27 \text{ cubic feet}\end{aligned}$$

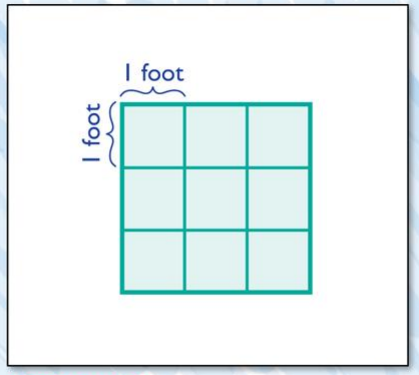

FIGURE 1.25 1 square yard is 9 square feet.

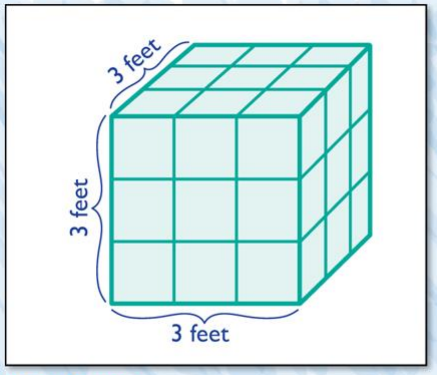

FIGURE 1.26 1 cubic yard is 27 cubic feet.

1.5 Critical thinking and number sense: What do these figures mean?

- **Example:** You want to redo your living room floor. Hardwood costs $10.40 per square foot, and the carpet you like costs $28.00 per (square) yard.

1. You need to decide right away whether you should be looking at hardwood or carpet for your floor. Use the fact that the cost of hardwood is about $10 per square foot to estimate the cost of a square yard of hardwood. Use your estimate to decide how the cost of hardwood compares with the cost of carpet.
2. Find the actual cost of a square yard of hardwood.

1.5 Critical thinking and number sense: What do these figures mean?

- **Solution:**

1. A common consumer error is to think that, because 1 yard is 3 feet, 1 square yard is 3 square feet. But

   1 square yard = 9 square feet

   Because hardwood costs about $10 per square foot, 9 square feet costs about $90. Because the carpet is $28 per yard, we estimate that hardwood is over three times as expensive as carpet.

2. To find the actual cost of a square yard of hardwood:

   $$\$10.40 \times 9 = \$93.60$$

   The result is a bit higher than the estimate of $90 we found in part 1, and it confirms our conclusion that hardwood is much more expensive than carpet.

Chapter 1 Critical Thinking: **Chapter Summary**

- Public policy and Simpson's paradox: Is "average" always average?
  - Understand that Simpson's paradox is a striking example of the need for critical thinking skills.
  - Overall average may lead to *invalid* conclusion.
- Logic and informal fallacies: Does that argument hold water?
  - Logical argument involves: Premises, Conclusion
  - Informal fallacies: *fallacies of relevance,*
    fallacies of presumption
  - Deductive arguments and Inductive arguments

Chapter 1 Critical Thinking: **Chapter Summary**

- Formal logic and truth tables: Do computers think?
  - Formal logic: The truth table
  - Operations on statements: Negation, conjunction, disjunction, conditional or implication.
- Sets and Venn diagrams: Pictorial logic
  - The Venn diagrams: Analyze logical statements.
- Critical thinking and number sense: What do these figures mean?
  - Relative sizes of numbers are indicated using *magnitudes* or powers of 10.
  - Estimation: To avoid complicated computations